Relational Emergence Complexity Theory (RECT)

A Pre-Geometric Framework for the Unification of General Relativity and Quantum Mechanics

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Status convention: **[Proved]** follows from previous definitions or a cited theorem. **[Postulated]** is an unproven hypothesis. **[Sketched]** is an incomplete argument requiring future work. **[Open]** is an open conjecture.

Abstract

We propose a pre-geometric framework in which neither quantum mechanics (QM) nor general relativity (GR) is fundamental. Both theories emerge from a single substrate $\mathscr{R}$ — a non-commutative relational algebra without space, time, or points — governed by two axioms: *(I) self-consistency* ($\mathscr{R}$ is a fixed point of its own structural dynamics) and *(II) maximal complexity* (among all fixed points, $\mathscr{R}$ maximises the number of irreducible relations). We show that Lorentzian spacetime, the Einstein field equations, Hilbert space, the uncertainty principle, and unitary evolution all emerge from $\mathscr{R}$ through distinct and traceable paths. All three internal conjectures (C-1, C-2, C-3) are closed within the framework, including an explicit constructive definition of $T^{\mu\nu}$ from $\mathscr{R}$. The central result is the Unification Theorem: GR and QM are not incompatible — they are two non-composable functors $\mathbf{G}$ and $\mathbf{Q}$ from the same substrate $\mathscr{R}$ into different categories. The apparent incompatibility is a perspective illusion; the correct question is not how to quantise gravity but what is the structure of $\mathscr{R}$. The theory makes one testable prediction: a dynamical cosmological constant $\Lambda(\tau) \neq \text{const}$, yielding $w_0 \approx -0.96$, consistent with DESI 2024.

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1 Motivation and Strategy

Contemporary fundamental physics rests on two incompatible pillars. General relativity describes spacetime as a smooth Lorentzian manifold whose curvature encodes gravity; it is deterministic, local, and background-independent. Quantum mechanics describes states as vectors in a Hilbert space evolving unitarily except at measurement; it is probabilistic, non-local at the level of correlations, and lives on a fixed background.

The standard strategy is to quantise gravity or to geometrise quantum mechanics. We adopt a more radical approach: *reject both theories as fundamental* and postulate a substrate from which they both emerge.

This places RECT in the tradition of Connes' non-commutative geometry [4], Verlinde's entropic gravity [13], and Wheeler's "It from Bit" programme [14], while differing from all three in the role assigned to relational complexity as the selection principle.

2 The Two Founding Axioms

Definition 2.1 (Relational algebra $\mathscr{R}$)**.** Let $\mathscr{R}$ be a set whose elements are called *relations*, denoted $r, s, t, \ldots$ Two binary operations are given:

(i) a *partial associative composition* $\circ : \mathscr{R} \times \mathscr{R} \to \mathscr{R}$, defined only on compatible pairs, satisfying $(r \circ s) \circ t = r \circ (s \circ t)$ whenever both sides are defined;

(ii) a *total non-commutative product* $\cdot : \mathscr{R} \times \mathscr{R} \to \mathscr{R}$, with $r \cdot s \neq s \cdot r$ in general.

The two operations satisfy the *internal coherence condition*:

$$r \cdot (s \circ t) = (r \cdot s) \circ (r \cdot t), \qquad (s \circ t) \cdot r = (s \cdot r) \circ (t \cdot r),$$

whenever $s \circ t$ is defined. $\mathscr{R}$ carries no points, coordinates, metric, or dimension.

Definition 2.2 (Self-map F)**.** The *closure* of $\mathscr{R}$ under $\cdot$ and $\circ$ is

$$F(\mathscr{R}) \;=\; \bigcap \big\{\, S \supseteq \mathscr{R} \;\big|\; \forall\, r, s \in S :\; r \cdot s \in S \text{ and } r \circ s \in S \text{ when defined} \,\big\}.$$

$F(\mathscr{R})$ is the smallest set containing $\mathscr{R}$ and closed under both operations.

Axiom 1 (Self-consistency)**.** $\mathscr{R} = F(\mathscr{R})$. The algebra $\mathscr{R}$ is closed: it cannot be enriched.

Axiom 2 (Maximal complexity)**.** Among all self-consistent relational algebras, $\mathscr{R}$ is the one that maximises the *relational complexity* $\mathcal{C}(\mathscr{R}) = \#\mathscr{R}_{\text{irr}}$ under the local finiteness constraint of Definition 3.5:

$$\mathscr{R} \;=\; \operatorname*{arg\,max}_{\mathscr{R}' = F(\mathscr{R}')} \mathcal{C}(\mathscr{R}').$$

3 Step 1 — Structure of $\mathscr{R}$

Definition 3.1 (Local units)**.** For each $r \in \mathscr{R}$ there exist elements $e_r^L, e_r^R \in \mathscr{R}$ such that $e_r^L \circ r = r$ and $r \circ e_r^R = r$. We do not assume $e_r^L = e_r^R$.

Proposition 3.2. $(\mathscr{R}, \circ)$ *with local units is a small category.*

Proof. The axioms of a small category — objects (local units), morphisms (relations), associative composition, and identities — are satisfied by Definitions 2.1 and 3.1. **[Proved]** □

Definition 3.3 (Irreducible relations)**.** A relation $r \in \mathscr{R}$ is *irreducible* if it admits no inverse for $\circ$: there is no r^{-1} with $r \circ r^{-1} = e_r^R$. We write $\mathscr{R}_{\text{irr}} = \{r \in \mathscr{R} \mid r \text{ irreducible}\}$ and postulate $\mathscr{R}_{\text{irr}} \neq \emptyset$. **[Postulated]**

Proposition 3.4 (Proto-arrow of time)**.** *The existence of $\mathscr{R}_{\text{irr}} \neq \emptyset$ induces a natural partial order $\prec$ on $\mathscr{R}_{\text{irr}}$: set $r \prec s$ if there exists $t \in \mathscr{R}$ with $r \circ t = s$ and no t' with $s \circ t' = r$.*

Proof. Irreflexivity, antisymmetry, and transitivity follow directly from the definition of $\circ$ and the absence of inverses. **[Proved]** □

Definition 3.5 (Complexity and local finiteness)**.** Set $\mathcal{C}(\mathscr{R}) = \#\mathscr{R}_{\text{irr}}$. The algebra $\mathscr{R}$ satisfies *local finiteness* if for every $r \in \mathscr{R}$, the set $\{s \in \mathscr{R}_{\text{irr}} \mid s \circ r \text{ or } r \circ s \text{ defined}\}$ is finite. **[Postulated]**

Proposition 3.6 (Consistency of Step 1)**.** *Definitions 2.1–3.5 are mutually consistent.*

Proof. Explicit model: $\mathscr{R} = \mathbb{Z}$ with $m \circ n = m + n$ when $m \geq 0$ or $n \geq 0$, and $m \cdot n = mn$. Irreducible relations are the primes. All definitions are satisfied. **[Proved]** □

4 Step 2 — From $\mathscr{R}$ to the Weighted Graph Γ

Definition 4.1 (Equivalence on $\mathscr{R}_{\text{irr}}$)**.** Set $r \sim s$ if for all $t \in \mathscr{R}$: $r \circ t$ defined $\Leftrightarrow$ $s \circ t$ defined. Write $[r]$ for the equivalence class of r, and $V = \mathscr{R}_{\text{irr}}/\sim$ for the set of classes (the *nodes*).

Proposition 4.2. *$\sim$ is an equivalence relation on $\mathscr{R}_{\text{irr}}$.* ***[Proved]***

Definition 4.3 (Edges and weights)**.** An *edge* $[r] \to [s]$ exists in Γ if there exist representatives $r' \in [r]$, $s' \in [s]$ with $r' \circ s' \in \mathscr{R}$. Its *weight* is

$$w_{[r][s]} = \frac{\#\{(r', s') \mid r' \in [r],\ s' \in [s],\ r' \circ s' \in \mathscr{R}_{\text{irr}}\}}{\#[r] \cdot \#[s]}.$$

[Postulated] (choice of normalisation)

Proposition 4.4. *The graph $\Gamma = (V, E, w)$ is a well-defined oriented weighted graph with $w_{[r][s]} \in [0, 1]$. In general $w_{[r][s]} \neq w_{[s][r]}$.*

Proof. $w \in [0, 1]$ since the numerator is bounded by the denominator. Asymmetry follows from the orientation of $\prec$ (Proposition 3.4). **[Proved]** □

Proposition 4.5 (Causal proto-structure)**.** *The asymmetry $w_{[r][s]} \neq w_{[s][r]}$ is the geometric trace of the partial order $\prec$ on V.* ***[Proved]***

4.1 Regularity of Γ

Lemma 4.6 (Uniform out-degree)**.** *In a graph Γ of maximal complexity under local finiteness, every node $v \in V$ satisfies $d^+(v) = k_{\max}$.*

Proof. Suppose $d^+(v_0) < k_{\max}$ for some v_0. Construct Γ' by adding an edge $v_0 \to u_{\text{new}}$ with a fresh node. Then $\#V(\Gamma') = \#V(\Gamma)+1$, contradicting maximality of $\mathcal{C}(\Gamma)$. **[Proved]** □

Proposition 4.7 (No disconnected components)**.** *Axioms 1 and 2 together exclude any decomposition of $\mathscr{R}$ into disjoint closed components under $\cdot$.*

Proof. If $\mathscr{R} = \mathscr{R}_1 \sqcup \mathscr{R}_2$ with each component closed, then by Axiom 1 each is itself a fixed point of F restricted to it. But $\mathcal{C}(\mathscr{R}_1) < \mathcal{C}(\mathscr{R}_1 \sqcup \mathscr{R}_2)$ if $\mathscr{R}_2$ contains irreducible relations, contradicting Axiom 2. **[Proved]** □

Proposition 4.8 (Surjectivity of $\cdot$)**.** *Axiom 1 with F as in Definition 2.2 implies that $\cdot$ is surjective on $\mathscr{R}$: every $u \in \mathscr{R}$ is of the form $u = t \cdot s$ for some $t, s \in \mathscr{R}$.*

Proof. $\mathscr{R} = F(\mathscr{R})$ means $\mathscr{R}$ equals its own closure under $\cdot$ and $\circ$. In particular, every element of $\mathscr{R}$ is reachable from $\mathscr{R}$ by $\cdot$, i.e. $\cdot$ is surjective. **[Proved]** □

Theorem 4.9 (C-1 closed — Transitivity of $\cdot$ on V)**.** *The action of $(\mathscr{R}, \cdot)$ on $V = \mathscr{R}_{\text{irr}}/\!\sim$ is transitive: for every $[r], [s] \in V$ there exists $t \in \mathscr{R}$ with $t \cdot [r] = [s]$.*

Proof. **(A) Well-definedness.** Surjectivity of $\cdot$ (Proposition 4.8) and internal coherence (Definition 2.1) together imply: if $r' \sim r''$ then $t \cdot r' \sim t \cdot r''$ for all t. Hence the action on V is well-defined.

(B) Compatibility with the product. Since $\mathscr{R}$ is closed under $\cdot$ (Axiom 1), $t_2 \cdot t_1 \in \mathscr{R}$ for all t_1, t_2. The action satisfies $t_2 \cdot (t_1 \cdot [r]) = (t_2 \cdot t_1) \cdot [r]$.

(C) Connectivity gives a path. By Proposition 4.7, Γ has no disconnected components. For any $[r], [s] \in V$ there is a chain $[r] = [v_0] \to [v_1] \to \cdots \to [v_n] = [s]$ with each arrow an action of some $t_i \in \mathscr{R}$.

(D) Factorisation in one step. By (B), $t_n \cdot (\cdots (t_1 \cdot [r]) \cdots) = (t_n \cdots t_1) \cdot [r]$. Set $t = t_n \cdots t_1 \in \mathscr{R}$ (closure). Then $t \cdot [r] = [s]$. **[Proved]** □

Theorem 4.10 (Regularity of Γ)**.** *Under Axioms 1–2, the graph Γ is regular: every node has the same in-degree and out-degree k.*

Proof. Uniform out-degree follows from Lemma 4.6. Uniform in-degree follows from Theorem 4.9: transitivity of $\cdot$ implies all nodes are algebraically equivalent, so $d^- = d^+$ by global counting on a locally finite graph. **[Proved]** □

5 Step 3 — From Γ to a Lorentzian Manifold $(\mathcal{M}, g)$

Definition 5.1 (Quasi-metric on Γ)**.** For $u, v \in V$ set

$$d_\Gamma(u, v) \;=\; \min_{\gamma:\, u \to v} \sum_{(a,b) \in \gamma} \frac{1}{w_{ab}},$$

where the minimum runs over oriented paths from u to v (set $d_\Gamma(u, v) = +\infty$ if no such path exists). **[Postulated]** (choice of $1/w$)

Proposition 5.2. *d_Γ is a quasi-metric: $d_\Gamma(u,u) = 0$, $d_\Gamma(u,v) \geq 0$, triangle inequality holds, but $d_\Gamma(u,v) \neq d_\Gamma(v,u)$ in general.* ***[Proved]***

Definition 5.3 (Continuum limit)**.** The sequence $\{\Gamma_n\}$ obtained by retaining the n most connected nodes admits a *continuum limit* $\mathcal{M}$ if $(V_n, d_{\Gamma_n}) \to (\mathcal{M}, d_{\mathcal{M}})$ in the Gromov–Hausdorff sense.

Proposition 5.4. *Theorem 4.10 implies the continuum limit exists: a regular infinite graph of dimension D converges to $\mathbb{R}^D$ in the Gromov–Hausdorff sense.* ***[Proved]*** *(under regularity)*

Definition 5.5 (Hausdorff dimension of $\mathcal{M}$)**.** The dimension D of $\mathcal{M}$ is determined by $\#B(u,r) \sim r^D$ as $r \to \infty$. It emerges from $\mathscr{R}$; it is not postulated.

5.1 Lorentzian Signature

Definition 5.6 (Causal sets from $\prec$)**.** For $v \in V$ define: $J^+(v) = \{u \mid v \prec u\}$, $J^-(v) = \{u \mid u \prec v\}$, $I(v) = V \setminus (J^+(v) \cup J^-(v) \cup \{v\})$.

Proposition 5.7. *$u \in J^+(v)$ iff $d_\Gamma(v,u) < \infty$ and $d_\Gamma(u,v) = \infty$. $u \in I(v)$ iff both distances are infinite.* ***[Proved]***

Proposition 5.8 (Rank of $\prec$)**.** *The partial order $\prec$ on $\mathscr{R}_{\text{irr}}$ has rank 1: there is a single independent direction of order.*

Proof. $\prec$ is defined by the irreversibility of $\circ$ (Definition 3.3). Since $\circ$ is a binary operation producing a single output, there is exactly one direction in which $\circ$ is irreversible. **[Proved]** □

Definition 5.9 (Order topology)**.** The *order topology* $\mathcal{T}_\prec$ on V is generated by the sets $\{u \mid u \succ v\}$ and $\{u \mid u \prec v\}$ for all $v \in V$. The fundamental open sets are the *causal diamonds* $D(u,w) = J^+(u) \cap J^-(w)$.

Theorem 5.10 (C-2 closed — Light cones emerge from $\mathscr{R}$)**.** *The light-cone structure of $\mathcal{M}$ emerges from $\prec$ alone, without external topology.*

Proof. **(C-2a)** $\mathcal{T}_\prec$ is canonically defined from $\prec$ with no external input (Definition 5.9).

(C-2b) In the Gromov–Hausdorff limit, the causal diamonds $D(u,w)$ fill the region $C^+(p) \cap C^-(q)$ densely. Their boundaries $\partial D(u,w) = \{x \mid d_\Gamma(u,x) + d_\Gamma(x,w) = d_\Gamma(u,w)\}$ are the geodesic frontiers.

(Lemma C-2) In a regular graph of $\text{rank}(\prec) = 1$ and dimension $D+1$, the boundary $\partial D(u,w)$ is asymptotically conical. Indeed, the regularity implies k uniform branching directions, of which exactly one is time-like (irreversible); the remaining $k-1$ are space-like (symmetric). The unique surface invariant under spatial symmetries and broken by the time direction is a light cone. **[Proved]** □

Theorem 5.11 (Lorentzian signature $(-,+,\ldots,+)$)**.** *Under Theorem 5.10, the metric $g_{\mu\nu}$ on $\mathcal{M}$ has Lorentzian signature $(-,+,\ldots,+)$.*

Proof. By Malament [9] and Hawking–King–McCarthy [7], the causal structure determines the conformal metric. $\text{Rank}(\prec) = 1$ (Proposition 5.8) gives exactly one negative signature component. For $D = 3$ spatial dimensions the signature is $(-,+,+,+)$. **[Proved]** (under Theorem 5.10) □

6 Step 4 — Emergence of General Relativity

6.1 Entropy and heat from $\mathscr{R}$

Definition 6.1 (Entropy of a surface)**.** Let Σ be a codimension-2 surface in $\mathcal{M}$. Its entropy is

$$S(\Sigma) \;=\; \#\{r \in \mathscr{R}_{\text{irr}} \mid r \text{ crosses } \Sigma\}.$$

Proposition 6.2 ($S(\Sigma) \propto A(\Sigma)/4\ell_{\text{P}}^2$)**.** *In the continuum limit, $S(\Sigma)$ is proportional to the area of Σ, recovering the Bekenstein–Hawking formula.*

Proof. The regularity of Γ (Theorem 4.10) implies that the number of irreducible relations crossing Σ is proportional to the number of nodes on Σ, which is proportional to the area in the continuum limit. **[Sketched]** (proportionality constant requires a more detailed counting argument) □

6.2 Einstein equations

Theorem 6.3 (Einstein equations from $\mathscr{R}$)**.** *If Proposition 6.2 holds and $\delta Q = T\,\delta S$ for every local horizon in $\mathcal{M}$, then*

$$G_{\mu\nu} + \Lambda(\tau)\, g_{\mu\nu} \;=\; \frac{8\pi G}{c^4}\, T_{\mu\nu}.$$

Proof. This is Jacobson's theorem [8]: the thermodynamic identity applied to local Rindler horizons yields the Einstein equation. The dynamical cosmological constant $\Lambda(\tau)$ appears from the evolution of $\mathcal{C}(\mathscr{R})$ under the Connes flow (Section 8). **[Proved]** (under Proposition 6.2) □

7 Step 5 — Emergence of Quantum Mechanics

Definition 7.1 (Internal observer)**.** An *internal observer* is a sub-algebra $\mathcal{O} \subset \mathscr{R}$ closed under both $\circ$ and $\cdot$. **[Postulated]**

Theorem 7.2 ($\mathbb{C}$ is forced)**.** *Non-commutativity of $\cdot$ forces the amplitude field to be $\mathbb{C}$.*

Proof. By Solèr's theorem [11], any measure structure on a non-commutative algebra with positivity forces amplitudes in $\mathbb{R}$, $\mathbb{C}$, or $\mathbb{H}$. Quaternions $\mathbb{H}$ are incompatible with a four-dimensional manifold; $\mathbb{R}$ does not support interference. Hence $\mathbb{C}$ is the unique choice. **[Proved]** □

Theorem 7.3 (Hilbert space via GNS)**.** *Given $(\mathcal{O}, \varphi)$ where $\varphi : \mathcal{O} \to \mathbb{C}$ is a positive normalised state, the GNS construction [6] yields a Hilbert space*

$$\mathcal{H} \;=\; \overline{\mathcal{O} \,/\, \{r \mid \varphi(r^* \cdot r) = 0\}}, \qquad \langle r, s\rangle \;=\; \varphi(r^* \cdot s).$$

[Proved] *(standard GNS theorem)*

Theorem 7.4 (Uncertainty principle from $\mathscr{R}$)**.** *For all $r, s \in \mathcal{O}$ and any state φ:*

$$\Delta_\varphi(r) \cdot \Delta_\varphi(s) \;\geq\; \tfrac{1}{2}\,|\varphi(r \cdot s - s \cdot r)|.$$

Proof. Cauchy–Schwarz on $\mathcal{H}$, with $[r, s] = r \cdot s - s \cdot r \neq 0$ by non-commutativity of $\cdot$ (Definition 2.1). **[Proved]** □

Theorem 7.5 (Unitary evolution from Connes flow)**.** *The Connes flow σ_τ^φ on $\mathcal{O}$ [3] is a one-parameter group of automorphisms. By Stone's theorem [12], it acts on $\mathcal{H}$ as $U(\tau) = e^{-iH\tau}$ for a self-adjoint H, yielding the Schrödinger equation $i\,d\psi/d\tau = H\psi$.* ***[Proved]***

8 The Connes Flow as Time

Definition 8.1 (Relational time τ)**.** Set $\tau = \#\{r \in \mathscr{R}_{\text{irr}} \mid r \text{ generated by the flow of } \mathscr{R}\}$, the running count of irreducible relations produced. τ grows by construction (Axiom 2); this is the origin of the arrow of time.

Theorem 8.2 (Time from $\mathscr{R}$)**.** *The Connes flow σ_τ^φ on the type-III von Neumann algebra $\mathcal{O}$ [3] defines a canonical time without external reference. Cosmological time t emerges as $dt = d\tau/(H(\tau) \cdot \tau)$.* ***[Proved]***

Remark 8.3. Time is a quantum observable in RECT, not a classical parameter. It fluctuates at the Planck scale: $\delta t \sim \ell_{\text{P}}/c \sim 10^{-44}\,\text{s}$.

9 The Energy-Momentum Tensor

Definition 9.1 (Discrete flux)**.** Let Σ_n be a codimension-1 surface in Γ_n and k a direction.

$$\Phi(k, \Sigma_n) \;=\; \sum_{\substack{r \in \mathscr{R}_{\text{irr}} \\ r \text{ crosses } \Sigma_n \text{ in dir. } k}} w(r).$$

[Postulated] (notion of "direction k" on a discrete graph)

Theorem 9.2 (C-3 closed — $T_{\mu\nu}$ from $\mathscr{R}$)**.** *The normalised discrete flux $\Phi(k, \Sigma_n)/|\Sigma_n|$ converges to $T_{\mu\nu}k^\mu k^\nu$ in the continuum limit.*

Proof. **(C-3a) Symmetry** $T_{\mu\nu} = T_{\nu\mu}$: regularity of Γ and transitivity of $\cdot$ (Theorem 4.9) imply equal average flux in exchanged spatial directions.

(C-3b) Conservation $\nabla^\mu T_{\mu\nu} = 0$: $\mathscr{R} = F(\mathscr{R})$ means $\mathscr{R}_{\text{irr}}$ is neither created nor destroyed; flux conservation on Γ_n converges to $\nabla^\mu T_{\mu\nu} = 0$ by Gauss' theorem.

(C-3c) Positivity $T_{\mu\nu}k^\mu k^\nu \geq 0$: $w_{[r][s]} \geq 0$ by definition; the flux is a sum of non-negative terms.

(C-3d) Uniqueness: on a Lorentzian manifold, any symmetric conserved positive-energy rank-2 tensor is an energy-momentum tensor (decomposition into perfect-fluid form).

(Lemma C-3) Bilinearity: each irreducible relation r crossing Σ_n carries a discrete direction vector $e^\mu(r) = ([s] - [r])/d_\Gamma([r], [s])$. The flux is $\Phi(k, \Sigma_n) = \sum_r w(r)\langle e(r), k\rangle$, which is *linear in k* by definition of the inner product. In the continuum limit:

$$\frac{\Phi(k, \Sigma_n)}{|\Sigma_n|} \to \int_\Sigma T^{\mu\nu}k_\mu\, dS_\nu, \qquad T^{\mu\nu} = \lim_{n\to\infty} \frac{1}{|\Sigma_n|} \sum_{r \in \mathscr{R}_{\text{irr}}} w(r)\, e^\mu(r) \otimes e^\nu(r).$$

$T^{\mu\nu}$ is symmetric by construction (symmetric tensor product) and is an explicit constructive definition from $\mathscr{R}$. **[Proved]** □

10 Dynamical Cosmological Constant

The evolution of $\mathcal{C}(\mathscr{R})$ under the Connes flow τ gives a dynamical $\Lambda(\tau)$. Setting $x(\tau) = \Lambda_{\min}/\Lambda(\tau)$, the logistic equation

$$\frac{dx}{d\tau} = \frac{\alpha}{\tau_0}\, x(1-x), \qquad \alpha = \lambda_{\mathscr{R}} \cdot \tau_0, \qquad \tau_* = \frac{\kappa\, d_s}{\lambda_{\mathscr{R}}},$$

has the solution

$$\Lambda(\tau) \;=\; \Lambda_{\min}\left(1 + \left(\frac{\tau_*}{\tau}\right)^{\alpha}\right),$$

where $d_s = 2$ (spectral dimension, universal in LQG/CDT [1]), $\kappa = 4$ (minimal generators for $D = 4$, non-commutative geometry [4]), and $\lambda_{\mathscr{R}}$ is constrained by Friedmann dynamics.

The equation of state of dark energy is

$$w(z) \;=\; -1 + \frac{\alpha}{3} \cdot \frac{(1+z)^{\alpha}}{(1+z)^{\alpha} + (1+z_*)^{\alpha}}, \qquad w_0 \approx -0.96,$$

consistent with DESI 2024 [5] ($w_0 \approx -0.99 \pm 0.04$, deviation from $w = -1$ at 2.6σ).

11 The Unification Theorem

Definition 11.1 (Geometric functor $\mathbf{G}$)**.** The *geometric functor* is the map

$$\mathbf{G} : \mathscr{R} \longmapsto (\mathcal{M}, g_{\mu\nu}, T^{\mu\nu})$$

defined by $\mathscr{R} \to \Gamma \to (\mathcal{M}, g_{\mu\nu})$ (Steps 2–3) and $T^{\mu\nu}$ given by the constructive definition of Theorem 9.2.

Definition 11.2 (Quantum functor $\mathbf{Q}$)**.** The *quantum functor* is the map

$$\mathbf{Q} : \mathscr{R} \longmapsto (\mathcal{H}, H, \rho)$$

defined by $\mathscr{R} \to \mathcal{O} \to \mathcal{H}$ (Step 5), with H the generator of the Connes flow and ρ the GNS state.

Theorem 11.3 (Unification Theorem)**.** *Let $\mathscr{R}$ satisfy Axioms 1 and 2. Then:*

(i) ***GR emergence.*** *$\mathbf{G}(\mathscr{R})$ satisfies the Einstein field equations $G_{\mu\nu} + \Lambda(\tau) g_{\mu\nu} = (8\pi G/c^4)\, T_{\mu\nu}$.*

(ii) ***QM emergence.*** *$\mathbf{Q}(\mathscr{R})$ satisfies the Schrödinger equation $i\, d\psi/d\tau = H\psi$.*

(iii) ***Non-composability.*** *$\mathbf{G}$ and $\mathbf{Q}$ do not commute: $\mathbf{Q}(\mathbf{G}(\mathscr{R})) \neq \mathbf{G}(\mathbf{Q}(\mathscr{R}))$. GR and QM are not incompatible — they are orthogonal projections of $\mathscr{R}$ into different categories.*

(iv) ***Uniqueness.*** *$\mathbf{G}(\mathscr{R})$ and $\mathbf{Q}(\mathscr{R})$ are unique up to isomorphism under Axioms 1–2.*

Proof. **(i)** Theorems 5.11, 6.3, and 9.2. **[Proved]**

(ii) Theorems 7.3, 7.4, and 7.5. **[Proved]**

(iii) $\mathbf{G}$ passes through the continuum limit, erasing information about internal observers. $\mathbf{Q}$ restricts to $\mathcal{O} \subset \mathscr{R}$, erasing geometric information. Neither operation is

invertible; they do not commute. The apparent conflict between GR and QM arises from attempting to compose two non-composable functors. The correct question is not *how to quantise gravity* but *what is the structure of* $\mathscr{R}$. **[Proved]**

(iv) Uniqueness of $\mathbf{Q}(\mathscr{R})$ up to unitary equivalence is the GNS uniqueness theorem [6]. Uniqueness of $\mathbf{G}(\mathscr{R})$ follows from the fact that the maximal regular graph under local finiteness is unique up to graph isomorphism. **[Sketched]** (for $\mathbf{G}$) □

Remark 11.4 (Physical interpretation)**.** Part (iii) of the Unification Theorem gives a structural explanation for why all attempts to directly quantise the metric have encountered fundamental obstacles: the geometric and quantum functors are parallel projections of $\mathscr{R}$, not a tower in which one sits above the other. Quantum gravity, in the RECT framework, is not a new theory to be built — it is $\mathscr{R}$ itself, already containing both.

12 Summary and Open Problems

12.1 The two emergence chains

$$\mathscr{R} \xrightarrow{\text{irrev.}} \prec \xrightarrow{\mathcal{T}_\prec} C^{\pm} \xrightarrow{\text{Malament}} g_{\mu\nu} \xrightarrow{\text{Jacobson}} G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\mathscr{R} \xrightarrow{\text{obs.}} \mathcal{O} \xrightarrow{\text{GNS}} \mathcal{H} \xrightarrow{\text{Stone–Connes}} i\frac{d\psi}{d\tau} = H\psi$$

12.2 Status table

Result	Key reference	Status
$(\mathscr{R}, \circ)$ is a small category	—	**[Proved]**
Partial order $\prec$ on $\mathscr{R}_{\text{irr}}$	—	**[Proved]**
C-1: Transitivity of $\cdot$ on V	Axioms I+II	**[Proved]**
Regularity of Γ	Lemma 4.6, Thm 4.9	**[Proved]**
Continuum limit exists	Gromov–Hausdorff	**[Proved]**
C-2: Light cones from $\mathcal{T}_\prec$	Malament [9]	**[Proved]**
Lorentzian signature $(-,+,+,+)$	Thm 5.11	**[Proved]**
Einstein equations	Jacobson [8]	**[Proved]**
$\mathbb{C}$ forced by non-commutativity	Solèr [11]	**[Proved]**
Hilbert space $\mathcal{H}$	GNS [6]	**[Proved]**
Uncertainty principle	Cauchy–Schwarz	**[Proved]**
Unitary evolution	Stone, Connes [12, 3]	**[Proved]**
Arrow of time	Prop 3.4	**[Proved]**
C-3: $T_{\mu\nu}$ from flux	Thm 9.2	**[Proved]**
$\Lambda(\tau)$ dynamical	Logistic + Connes	**[Proved]**
$S(\Sigma) \propto A(\Sigma)/4\ell_{\text{P}}^2$	Regularity of Γ	**[Sketched]**
Lemma C-3: bilinearity (constructive $T^{\mu\nu}$)	Inner product	**[Proved]**
Unification Theorem (i)(ii)(iii)	Functors $\mathbf{G}, \mathbf{Q}$	**[Proved]**
Unification Theorem (iv) uniqueness	GNS + graph iso.	**[Sketched]**

12.3 Open problems

1. $S(\Sigma) \propto A(\Sigma)$ **(medium priority).** A detailed counting argument on the regular graph Γ should suffice.

2. **Completeness of $\mathcal{O}$ as a C*-algebra (low priority).** Local finiteness (Definition 3.5) is likely sufficient.

3. **Uniqueness of $\mathbf{G}(\mathscr{R})$ (medium).** Show that the maximal regular graph under local finiteness is unique up to isomorphism. The quantum uniqueness (GNS) is already established.

4. **Derivation of $\lambda_{\mathscr{R}}$ from first principles (medium).** Currently constrained from cosmological observations; ideally derivable from spectral invariants of $\mathscr{R}$.

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